Purdue University
Purdue e-Pubs

Publications of the Ray W. Herrick Laboratories | School of Mechanical Engineering

5-21-2013

The Application of Singular Value Decomposition to Determine the Sources of Far Field Diesel Engine Noise

J Stuart Bolton
Purdue University, bolton@purdue.edu

Patricia Davies
Purdue University, davies@purdue.edu

Michael D. Hayward
Purdue University

Follow this and additional works at: http://docs.lib.purdue.edu/herrick

Bolton, J Stuart; Davies, Patricia; and Hayward, Michael D., "The Application of Singular Value Decomposition to Determine the Sources of Far Field Diesel Engine Noise" (2013). *Publications of the Ray W. Herrick Laboratories.* Paper 102.
http://docs.lib.purdue.edu/herrick/102

This document has been made available through Purdue e-Pubs, a service of the Purdue University Libraries. Please contact epubs@purdue.edu for additional information.

THE APPLICATION OF SINGULAR VALUE DECOMPOSITION TO DETERMINE THE SOURCES OF FAR FIELD DIESEL ENGINE NOISE

Michael D. Hayward
Patricia Davies
J. Stuart Bolton

May 21 2013
SAE Noise and Vibration Conference
Grand Rapids, Michigan

Introduction

- Demand for quieter engines is a constant driving force behind creation of competitive engines
- Determination of dominant noise sources in both the near- and far-fields within an engine is an integral step in development of quieter engines
- A method to assist in noise source identification in the near- and far-fields was desired to reduce the number of time-consuming and expensive fired and motored tests required

Outline

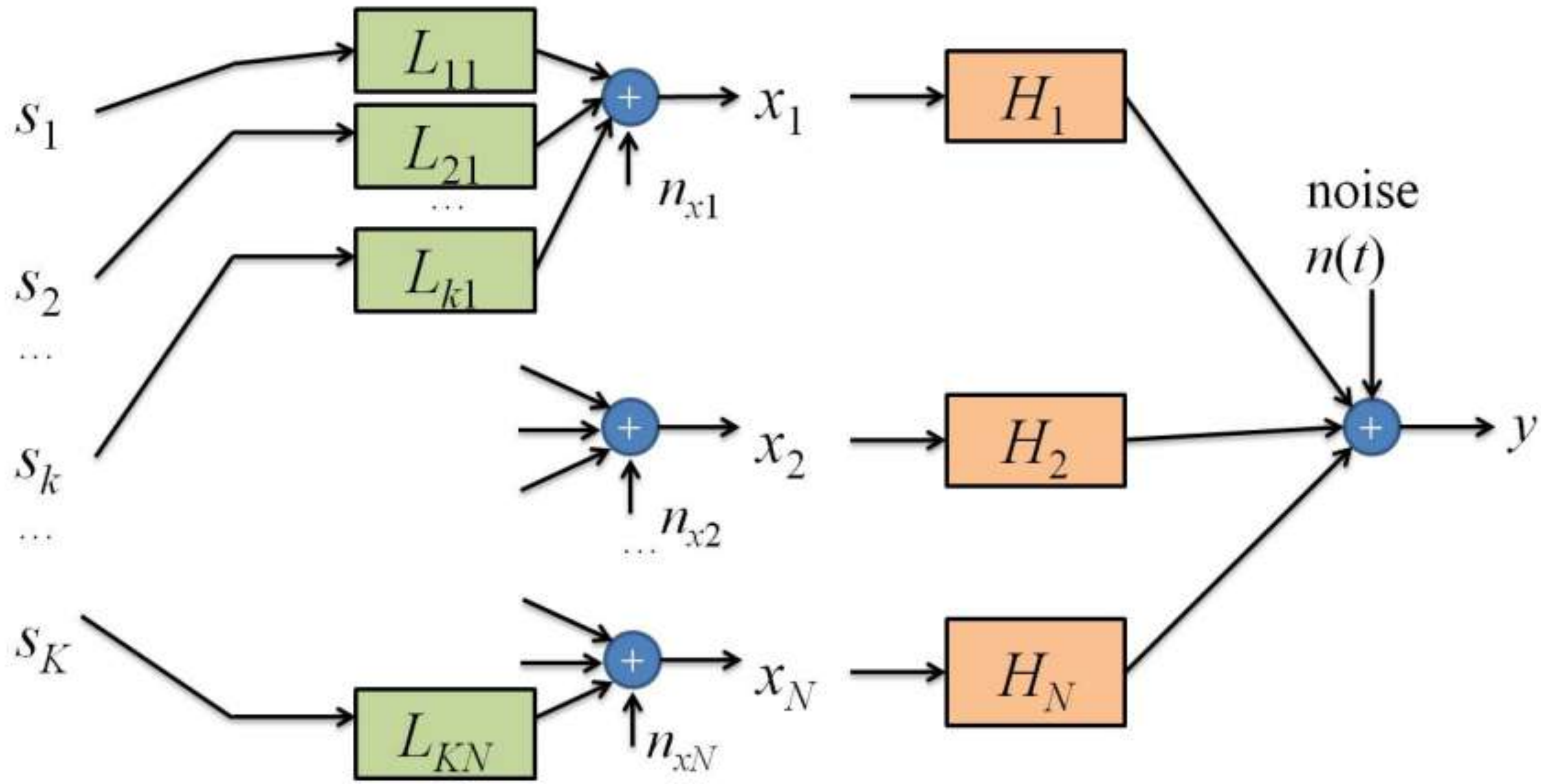
Multiple Input/Multiple Output (MIMO) System

Transfer Path Estimation Between Inputs and Outputs

Singular Value Decomposition

SVD Contributions to Near- and Far-Field Measurements

Multiple Input/Single Output System



True, independent sources (not measured)

Input Near Field Measurements (Microphones, Accelerometers, etc.)

Transfer Paths

Output Far Field Measurements (Microphones)

Solution of Cross-Spectral Matrix Problem

- A method to calculate transfer paths without including repeated information was required.
- In the following simplified equation, H_1 and H_2 are needed

$$\begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} H_1 \\ H_2 \end{bmatrix} = \begin{bmatrix} S_{1y} \\ S_{2y} \end{bmatrix}$$

Well Conditioned S_{xx} Matrix

Transfer Paths

S_{xy} Matrix

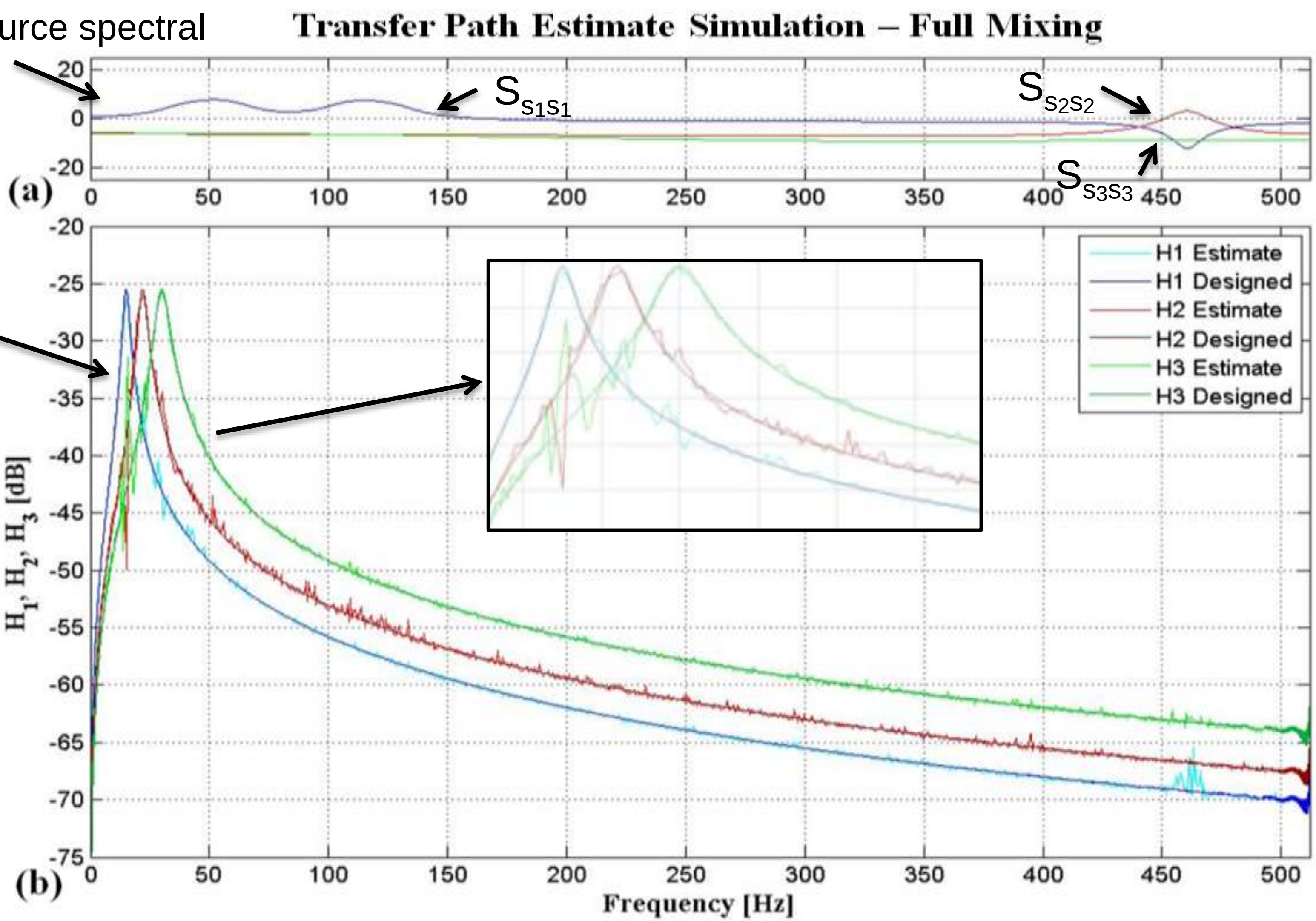
- Solution of this matrix by elementary row operations (Gaussian elimination) was conducted;

$$\begin{bmatrix} S_{11} & S_{12} \\ 0 & S_{22} - \dfrac{S_{12}S_{21}}{S_{11}} \end{bmatrix} \begin{bmatrix} H_1 \\ H_2 \end{bmatrix} = \begin{bmatrix} S_{1y} \\ S_{2y} - \dfrac{S_{1y}S_{21}}{S_{11}} \end{bmatrix}$$

Transfer Path Estimation

Independent source spectral characteristics

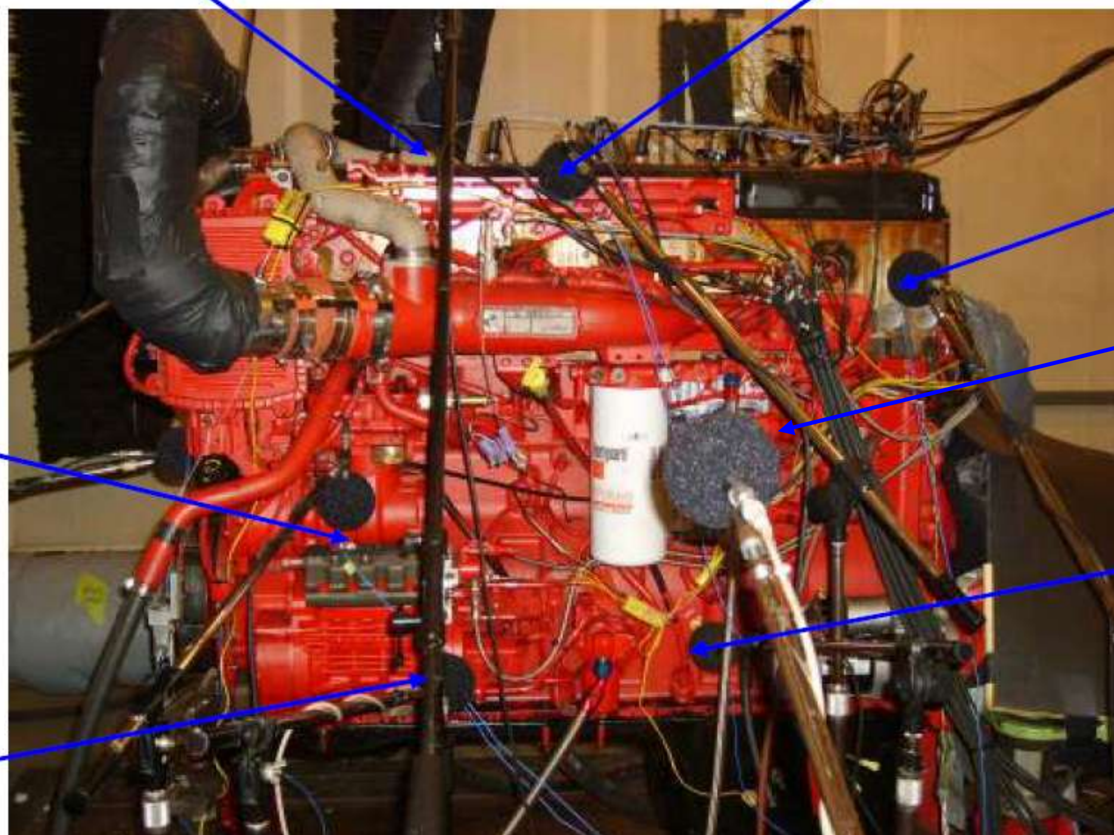
Designed and estimated transfer paths



Testing of Engine

Measurement

Measurement



Measurement

1 Meter
Microphone
(Output)

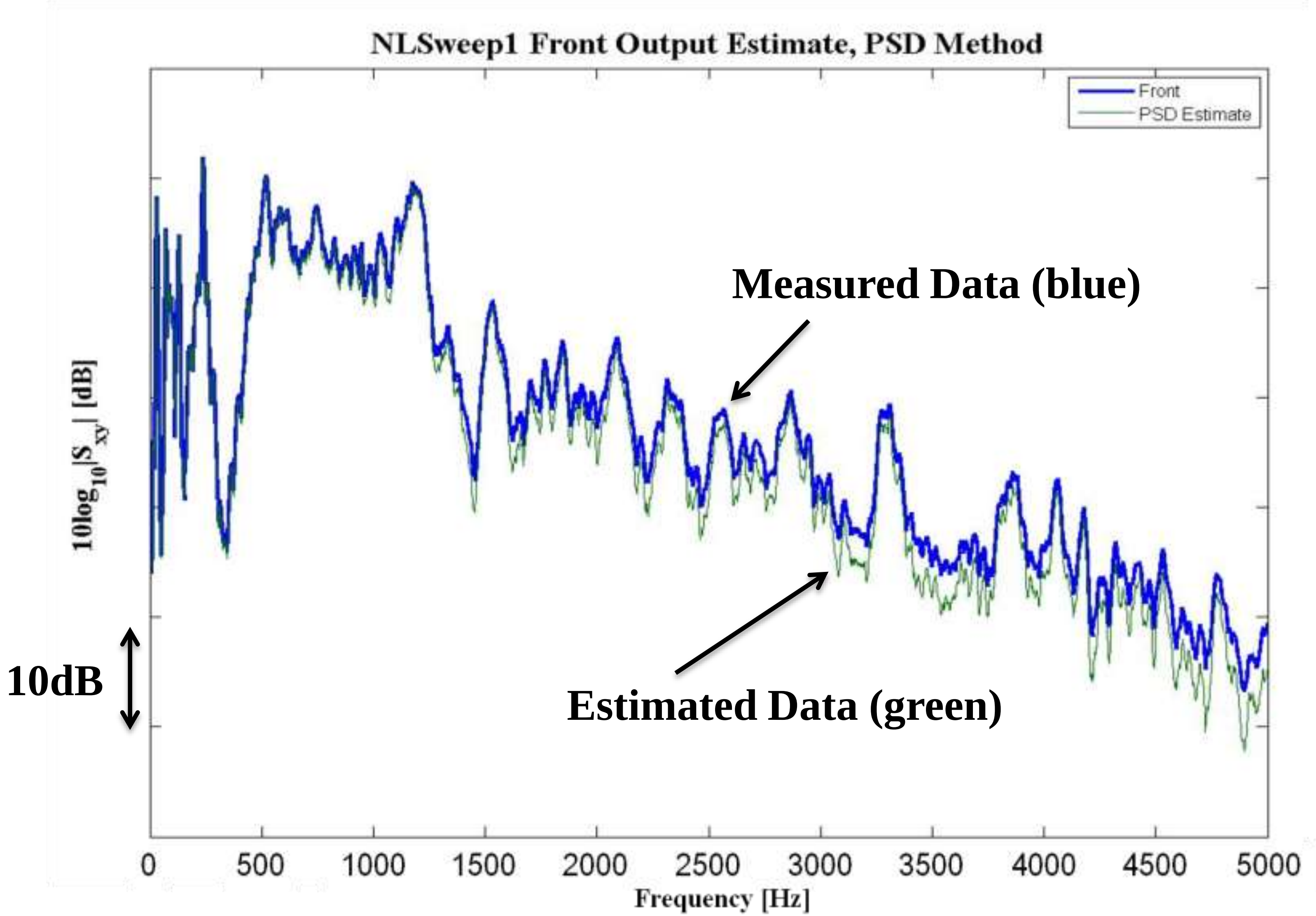
Measurement

Measurement

Measurement

- Each near-field measurement is an accelerometer/near-field microphone pair
- There are also 6 cylinder pressure transducers and 4 far-field microphones

Far Field Estimate Power Spectra



Singular Value Decomposition

Separate noise from uncorrelated sources from the data measured by the inputs

$$[\mathbf{S}_{\mathbf{xx}}] = \mathbf{U}\Sigma\mathbf{V}^{\mathbf{H}} = [\mathbf{u}_{\mathbf{1}}, \mathbf{u}_2, .. \mathbf{u}_N]\, diag[\lambda_1, \lambda_2, ... \lambda_N][\mathbf{v}_{\mathbf{1}}, \mathbf{v}_2, .. \mathbf{v}_N]^H$$

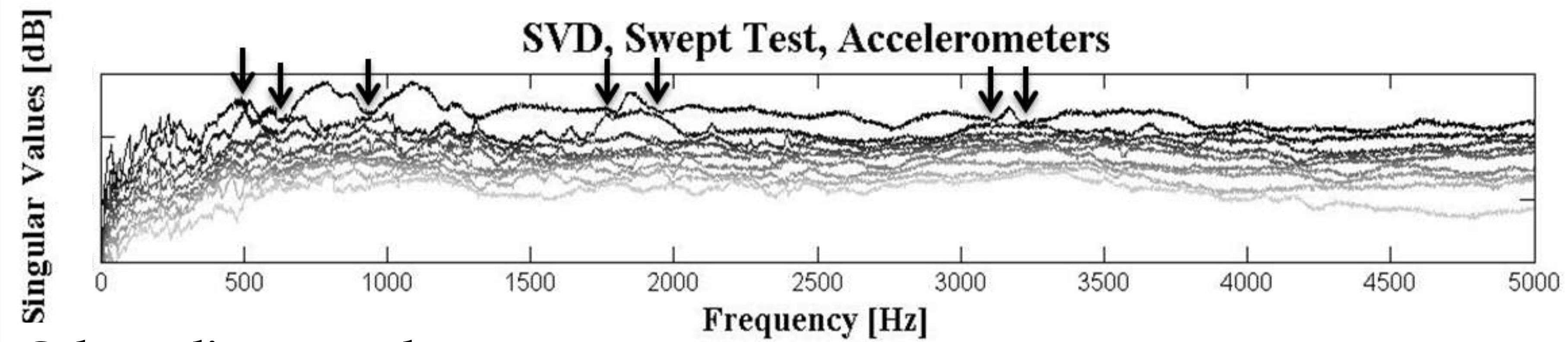
$$U = V \qquad \lambda_1 > \lambda_2 > ... > \lambda_N$$

[S$_{xx}$]= Cross spectral matrix
N = Number of input measurements to system
λ_i = Singular value
$\mathbf{u_i}$ = Left singular vector
$\mathbf{v_i}$ = Right singular vector
u = **v** in this case

Conducting a decomposition on the accelerometer or near field microphone spectral density matrices will help us identify the number of uncorrelated sources being measured, and potentially their relative strength.

Golub G H and Loan C F V 1996 *Matrix Computations*, 3rd Edition, (Baltimore: Johns Hopkins University Press).

SVD Example and Decomposition



Color coding example

1st Singular Value Contribution

2nd Singular Value Contribution

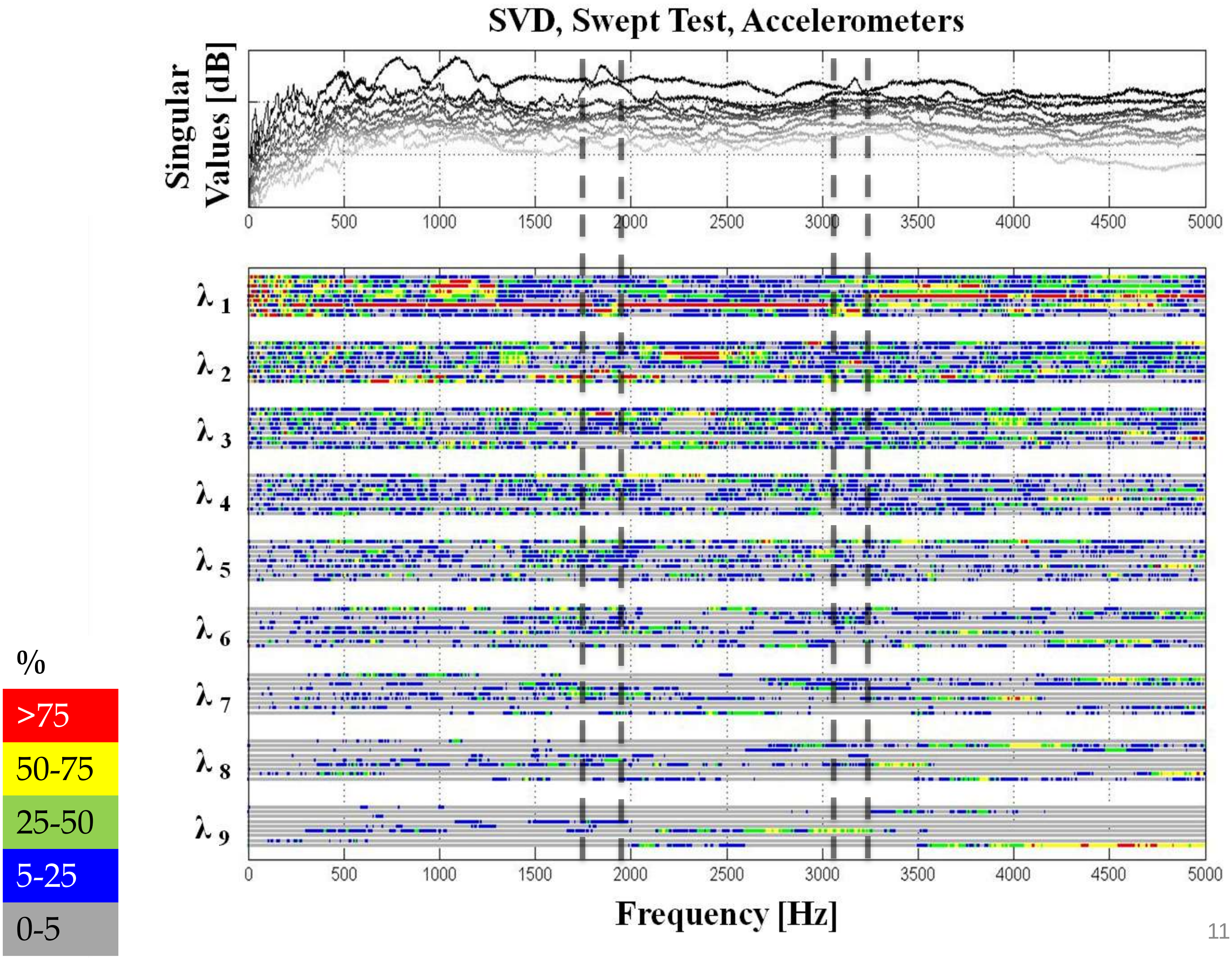
nth Singular Value Contribution

$$S_{x_1x_1} = \underbrace{\overbrace{\mathbf{u}_1\lambda_1\mathbf{v}_1^H}}_{8\%} + \underbrace{\overbrace{\mathbf{u}_2\lambda_2\mathbf{v}_2^H}}_{78\%} + \ldots + \overbrace{\mathbf{u}_n\lambda_n\mathbf{v}_n^H}$$

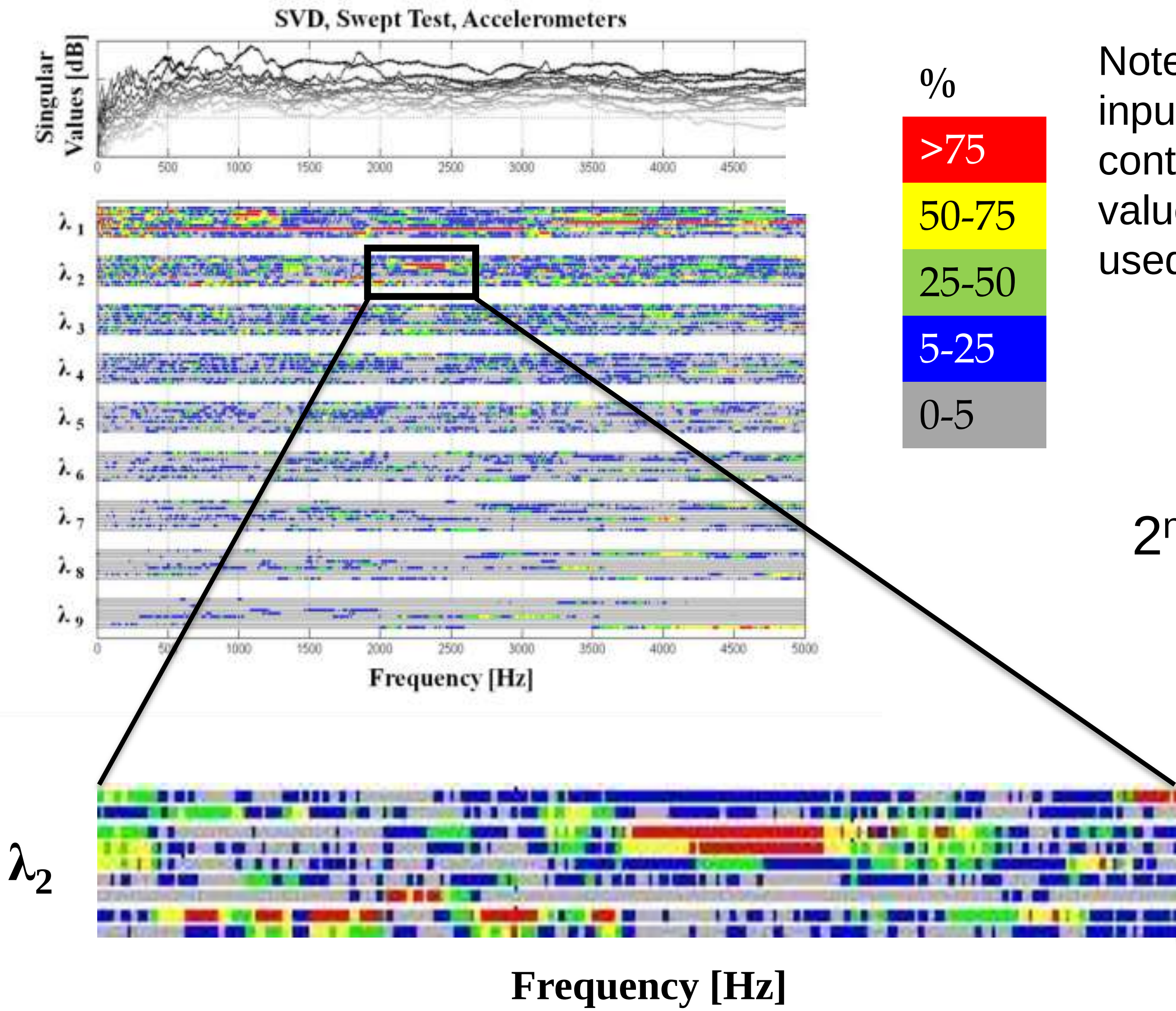
%
>75
50-75
25-50
5-25
0-5

- Using a color coding scheme depending on the percentage contribution, these can be visualized graphically.

- A more detailed description of this method can be found in Hayward, Bolton & Davies (2012).



SVD, Swept Test, Accelerometers
Singular Values [dB]
0
500
1000
1500
2000
2500
3000
3500
4000
4500
5000
λ 1
λ 2
λ 3
λ 4
λ 5
λ 6
λ 7
λ 8
λ 9
Frequency [Hz]
%
>75
50-75
25-50
5-25
0-5



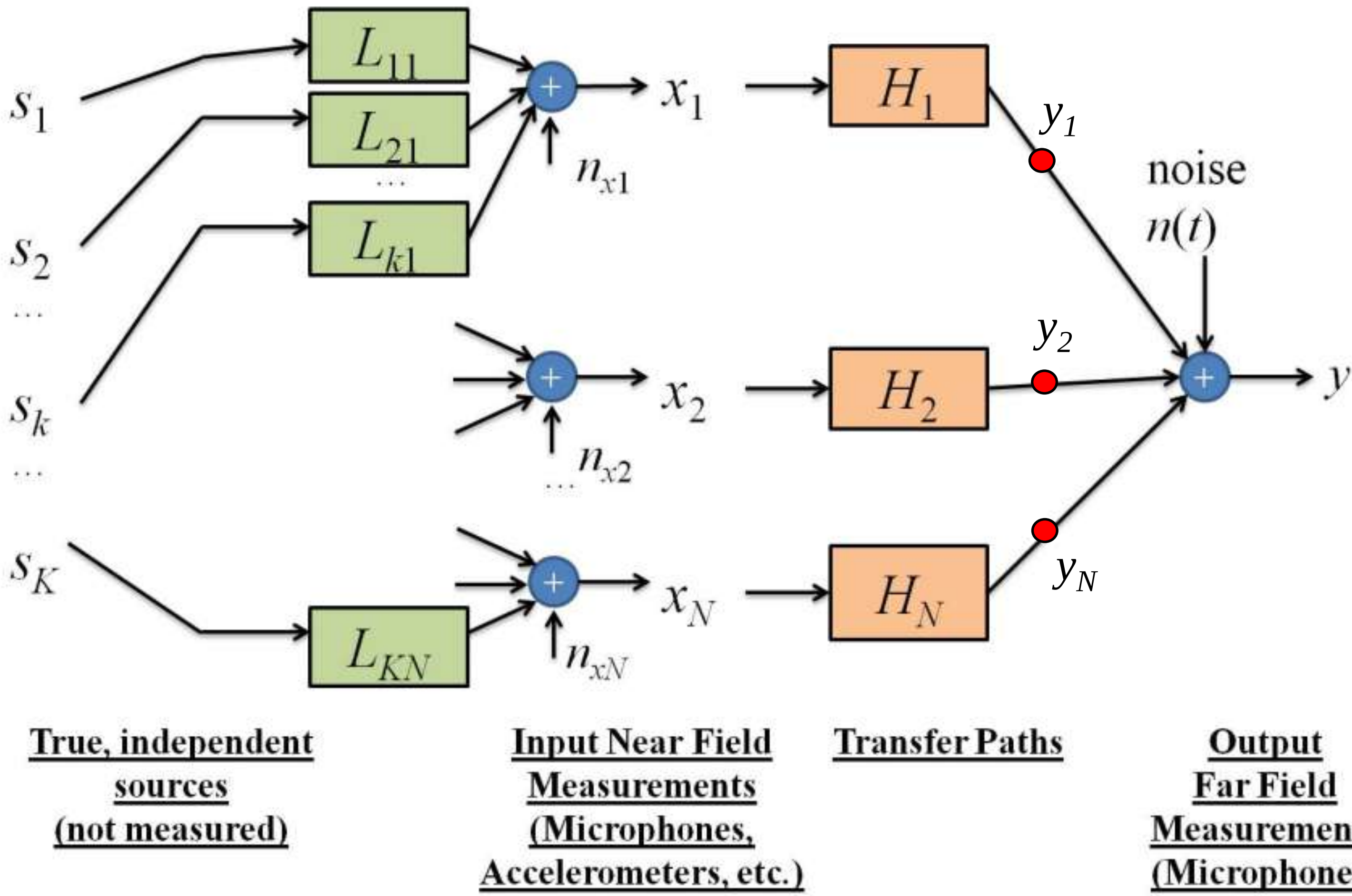
%
>75
50-75
25-50
5-25
0-5

Note: This analysis uses input measurement data – contribution of singular values at output is not used

2nd Singular Value Contribution to:

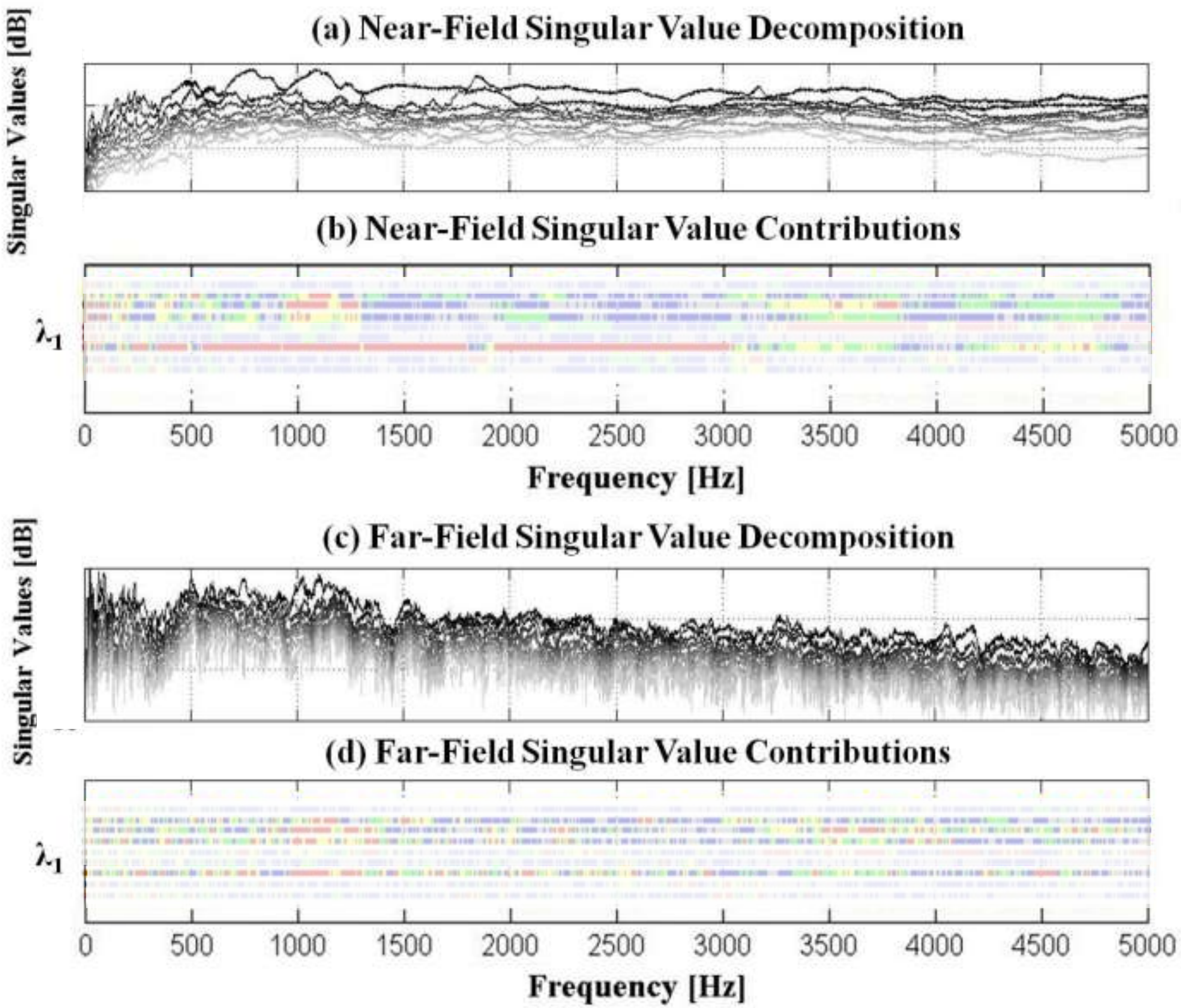
Measurement 1
Measurement 2
Measurement 3
Measurement 4
Measurement 5
Measurement 6
Measurement 7
Measurement 8
Measurement 9

Far Field Source Contributions



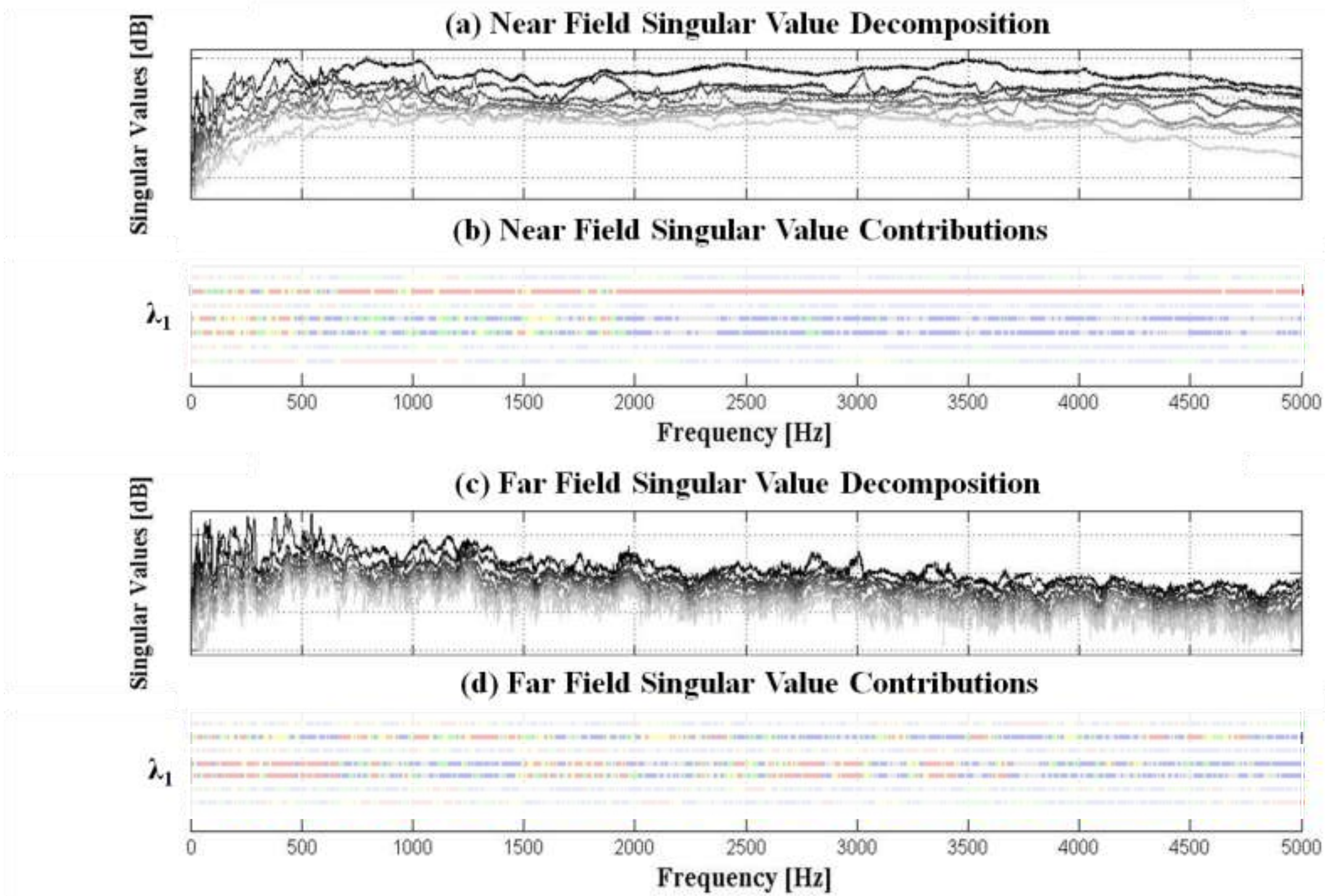
With calculated transfer paths, far-field time histories can be calculated, and source contributions to the far field can be determined

Near-Field and Far-Field Contribution Comparison Example 1



Near-Field and Far-Field Contribution Comparison Example 2

The analysis below is was conducted on data from a different engine, in a full loaded sweep test, with a different set of transducers



Conclusions

- Transfer paths from input to output measurements can be accurately estimated through solution of a cross-spectral matrix problem.
- Contributions of each independent virtual source to real, physical near-field locations can be determined through singular value contribution plots.
- Utilization of input measurements and estimated transfer paths yield both accurate far-field estimate time histories, and SVD contribution plots demonstrating virtual source contributions in the far-field.
- Analysis of the singular values and their contributions to near- and far-field measurement power spectra allows inferences to be drawn regarding the characteristics of dominant noise sources within the engine.

References

Otte D, Sas P and Van de Ponseele 1988 *Noise Source Identification by use of Principal Component Analysis*, Proceedings of Inter-Noise 88 (France: Anvignon)

Kompella MS, Ufford DA, Davies P and Bernhard RJ 1996 *A technique to determine the number of incoherent sources contributing to the response of a system*, Mechanical Systems and Signal Processing, vol. 8 no. 4 pp. 363-380

Leclère Q, Pèzerat C, Laulagnet B and Polac L 2005 *Application of Multi-Channel Spectral Analysis to Identify the Source of a Noise Amplitude Modulation in a Diesel Engine Operating at Idle*, Applied Acoustics, vol. 66 no. 7 pp. 779-798

Hayward, M.D., Bolton, J.S., and Davies, P. 2012 *Connecting the singular values of an input cross-spectral density matrix to noise sources in a diesel engine*, INTER-NOISE and NOISE-CON Congress and Conference Proceedings, Vol. 2012, no. 2, pp. 9583-9593, Institute of Noise Control Engineering

<u>Singular Value Decomposition</u>

Golub G H and Loan C F V 1996 *Matrix Computations*, 3rd Edition, (Baltimore: Johns Hopkins University Press).